Marking Scheme

SUMMATIVE ASSESSMENT – II (2015-16)

Mathematics (Class – X)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity and maintain uniformity. The answers given in the marking scheme are the best suggested answers.
2. Marking be done as per the instructions provided in the marking scheme. (It should not be done according to one's own interpretation or any other consideration).
3. Alternative methods be accepted. Proportional marks be awarded.
4. If a question is attempted twice and the candidate has not crossed any answer, only first attempt be evaluated and 'EXTRA' be written with the second attempt.
5. In case where no answers are given or answers are found wrong in this Marking Scheme, correct answers may be found and used for valuation purpose.

/ SECTION-A

141 Question numbers **1** to **4** carry **one** mark each.

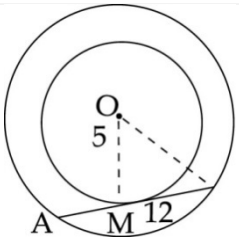
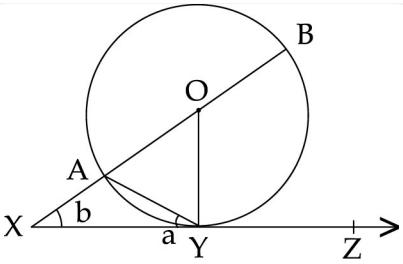
1	$l - d$	1
2	$30°$	1
3	$\frac{1}{4}$	1
4	2	1

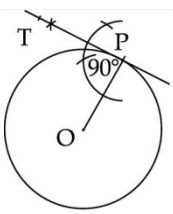
/ SECTION-B

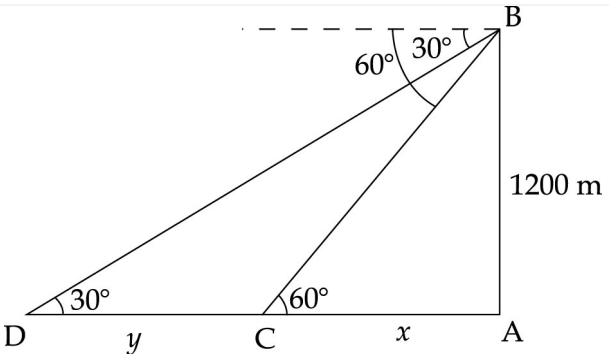
5 10 2

Question numbers **5** to **10** carry **two** marks each.

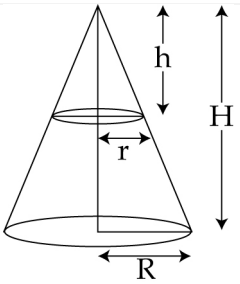
5	$D = b^2 - 4ac$ $D = (-3a)^2 - 4 \times 4 \times 1$ $9a^2 = 16$ or $a = \pm \frac{4}{3}$	2
6	Let 1st term be a and common diff be d $a_{32} = 2a_{12}$ $a + 31d = 2\,(a + 11d)$ $a + 31d = 2\,a + 22d$ $a = 9d$ $a_{70} = a + 69d = 9d + 69d = 78\,d$ $a_{31} = a + 30d = 9d + 30d = 39\,d$	2

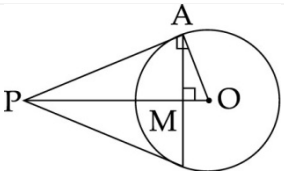
		$\therefore a_{70} = 2a_{31}$ Proved	
	7	 Join OM, OB OM $\perp$ AB (radius $\perp$ tgt) $\therefore$ MB $= \frac{1}{2}$ AB ($\perp$ from centre bisects chord) $= 12$ In rt Δ OMB, $OB^2 = OM^2 + MB^2$ $= 5^2 + 12^2$ $= 25 + 144$ $= 169$ OB = 13 cm Radius of larger circle = 13 cm	2
	8	Construction with proper labelling	2
	9	 In Δ AXY, ext $\angle YAO = \angle AXY + \angle AYX$ (ext angle theorem) $= b + a$ But $\angle OAY = \angle OYA$ (angles opp equal sides which are radii) $= a + b$ $\therefore$ But OY $\perp$ XZ (radius $\perp$ tgt) $\therefore \angle OYX = 90°$ or $\angle OYA + \angle AYX = 90°$ ie $a + b + a = 90°$ or $2a + b = 90°$	2
	10	Circumference of a circle $= 2\pi r_1$ $r_1 = 19$ cm and $r_2 = 9$ cm Sum of the two circumferences of two circles $= 2\pi r_1 + 2\pi r_2$ $= 2\pi \times 19 + 2\pi \times 9$	2

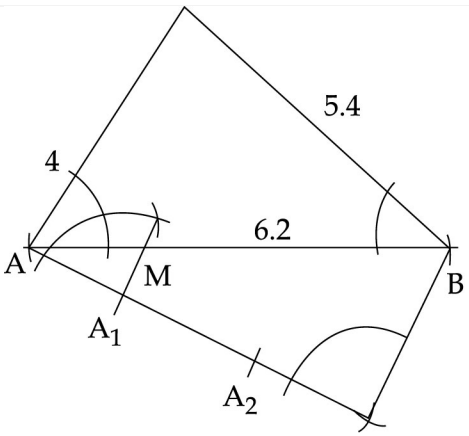
		$= 56\pi$ cm Let the radius of required circle is 'R' Then $2\pi R = 56\pi$ $\Rightarrow$ R = 28 cm.	
		/ SECTION-C	
		11 203 Question numbers **11** to **20** carry **3** marks each.	
	11	$2kx^2 - 40x + 25 = 0$ $a = 2k,\ b = -40,\ c = 25$ For equal roots ½ $D = 0$ $b^2 - 4ac = 0$ $(-40)^2 - 4 \times 2k \times 25 = 0$ $1600 - 200k = 0$ 1 $k = 8$ $16x^2 - 40x + 25 = 0$ $\Rightarrow \quad (4x-5)^2 = 0$ 1 $\Rightarrow \quad 4x = 5$ $x = \frac{5}{4}$ $\therefore$ Roots are $\frac{5}{4}$ and $\frac{5}{4}$ ½	3
	12	Given, $a_3 = 9 \quad \Rightarrow \quad a + 2d = 9$ (1) $a_8 - a_5 = 6$ $\Rightarrow \quad (a+7d) - (a+4d) = 6$ $3d = 6$ $d = 2$ (2) $\Rightarrow \quad a + 2(2) = 9$ Substituting (2) in (1) $a = 5$ So, A.P = 5, 7, 9, 11 . . .	3
	13	 Join OP Const 90^0 angle at P to OP PT is tangent Neatness	3

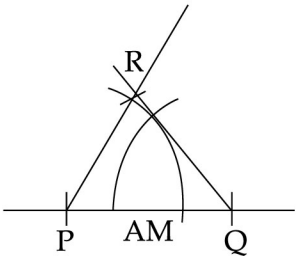
		PT is tangent to the circle at P.	
	14	 Let the aeroplane be at B and the two ships be at C and D. Let AC = x and CD = y In ΔCAB, $\tan 60° = \frac{AB}{AC}$ $\Rightarrow \quad \sqrt{3} = \frac{1200}{x} \quad \Rightarrow \quad x = \frac{1200}{\sqrt{3}}$ $\Rightarrow \quad x = 400\sqrt{3}$ m In ΔBAD, $\tan 30° = \frac{AB}{AD}$ $\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1200}{x+y}$ $\Rightarrow \quad x + y = 1200\sqrt{3}$ $\Rightarrow \quad y = 1200\sqrt{3} - x$ $= 1200\sqrt{3} - 400\sqrt{3}$ $= 800\sqrt{3}$ m	3
	15	P(not red) No. of pens which are not red = 80 $P(\text{Not red}) = \frac{80}{100} = \frac{4}{5}$ No. of green pens = 35 $P(\text{green}) = \frac{35}{100} = \frac{7}{20}$	3
	16	Let P($3x-8$, $2x-5$) be the centre of the circle passing through the given point A ($+4$, $+3$). The length of diameter is $4\sqrt{13}$ units $\therefore \quad$ Radius $= 2\sqrt{3}$ $\Rightarrow \quad$ PA $= 2\sqrt{13}$ $\Rightarrow \quad PA^2 = \left(2\sqrt{13}\right)^2$ $\Rightarrow \quad (3x-8-4)^2 + (2x-5-3)^2 = (4 \times 13)$ $\Rightarrow \quad (3x-12)^2 + (2x-8)^2 = 52$ $\Rightarrow \quad 9x^2 + 144 - 72x + 4x^2 + 64 - 32x = 52$	3

		$\Rightarrow \quad 13x^2 - 104x + 208 - 52 = 0$ $\Rightarrow \quad 13x^2 - 104x + 156 = 0$ $\Rightarrow \quad x^2 - 8x + 12 = 0$ $\Rightarrow \quad x^2 - 6x - 2x + 12 = 0$ $\Rightarrow \quad x(x-6) - 2(x-6) = 0$ $\Rightarrow \quad (x-2)(x-6) = 0$ $\Rightarrow \quad (x-2) = 0$ or $(x-6) = 0$ $\Rightarrow \quad x = 2$ or $x = 6$ Hence, the value of x is either 2 or 6	
	17	Let the third vertex be (x, y). centroid $= (-7, 4)$ (given) With vertices (22, 20), $(-2, -3)$ and (x, y) $\therefore \quad \frac{22 + (-2) + x}{3} = -7$ $\Rightarrow \quad 20 + x = -21$ $\Rightarrow \quad x = -21 - 20$ $\Rightarrow \quad x = -41$ And $\frac{20 + (-3) + y}{3} = 4$ $\Rightarrow \quad 17 + y = 12$ $\Rightarrow \quad y = 12 - 17$ $\Rightarrow \quad y = -5$ $\therefore$ The third vertices is $(-41, -5)$.	3
	18	Area of circle $= \pi r^2$ $22176 = \frac{22}{7} \times r^2$ $r = 84$ m circumtrance $= 2\pi r = 528$ m cost of fencing $= 528 \times 50 =$ ₹ 26400	3
	19	Here $L = 2b$ Let breadth be x cm, Length $= 2x$ cm. Area of 4 sides $= 2h(l + b)$ $1440 = 2(10)(2x + x)$ $1440 = 20(3x)$ $1440 = 60x$ $\frac{1440}{60} = x$ $24 = x =$ breadth $\therefore$ Length $= 48$ cm Volume $= l \times b \times h = 48 \times 24 \times 10$ $= 11520$ cm^3.	3

20	 Let the height of cone = H cm. Let it be divided into two equal parts of x cm each. Since triangles are similar therefore, $\frac{r}{R}=\frac{h}{H}\Rightarrow\frac{r}{R}=\frac{x}{2x}\Rightarrow\frac{r}{R}=\frac{1}{2}\Rightarrow R=2r$ Now $\frac{\text{Volume of small cone}}{\text{Volume of frustum}}=\frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(R^2+r^2+Rr\right)H}$ $\Rightarrow \frac{\text{Volume of small cone}}{\text{Volume of frustum}}=\frac{\frac{1}{3}\pi r^2 x}{\frac{1}{3}\pi\left(4r^2+r^2+2r^2\right)x}$ $\Rightarrow \frac{\text{Volume of small cone}}{\text{Volume of frustum}}=\frac{r^2}{7r^2}$ $\Rightarrow \frac{\text{Volume of small cone}}{\text{Volume of frustum}}=\frac{1}{7}$	3
	/ SECTION-D	
	21314 Question numbers **21** to **31** carry **4** marks each.	
21	$\frac{(4x-3)^2-10(2x+1)^2}{(2x+1)(4x-3)}=3$ $\Rightarrow (16x^2-24x+9)-10(4x^2+4x+1)=3(2x+1)(4x-3)$ $\Rightarrow 16x^2-24x+9-40x^2-40x-10=3(8x^2-6x+4x-3)$ $\Rightarrow -24x^2-64x-1=24x^2-18x+12x-9$ $\Rightarrow 48x^2+58x-8=0$ $\Rightarrow 24x^2+29x-4=0$ $\Rightarrow 24x^2+32x-3x-4=0$ $\Rightarrow (3x+4)(8x-1)=0$ $\Rightarrow 3x+4=0 \quad \vert \quad 8x-1=0$ $x=\frac{-4}{3} \quad \vert \quad x=\frac{1}{8}$	4
22	10, 20, 30, ……., 1000 (numbers divisible by 2 and 5) $a=10,\ d=10,\ a_n=1000$	4

		$a+(n-1)d=1000 \quad \Rightarrow n=100$ $S_{100}=\frac{100}{2}(10+1000)=50500$ Sum of first 1000 natural numbers $=\frac{1000}{2}(1+1000)=500\times1001$ $=500500$ $\therefore$ Reqd. sum $=500500-50500$ $=450000$	
	23	$\left(\frac{2x}{x-5}\right)^2+5\left(\frac{2x}{x-5}\right)-24=0$ Let $\frac{2x}{(x-5)}=y$ $\therefore y^2+5y-24=0$ $(y+8)(y-3)=0$ $y=3, -8$ $y=3$ $x=15$ $y=-8$ $x=4$	4
	24	 In ΔPAO and ΔAMO (i) $\angle O=\angle O$ (common) (ii) $\angle PAO=\angle AMO$ (each 90°) (iii) $\angle OPA=\angle OAM$ (third angle)	4
	25	Scale factor $\frac{1}{3}$ means $\frac{PQ}{AB}=\frac{PR}{AC}=\frac{QR}{BC}=\frac{1}{3}$	4





Drawing ΔABC

Marking M on AB so that

$\frac{AM}{BC}=\frac{1}{3}$

Drawing ΔPQR on PQ= AM

$\angle P=\angle A$, $\angle Q=\angle B$

$\angle P=\angle A$, $\angle R=\angle B$ and PR=AM

$\therefore \Delta PQR \sim \Delta ABC$

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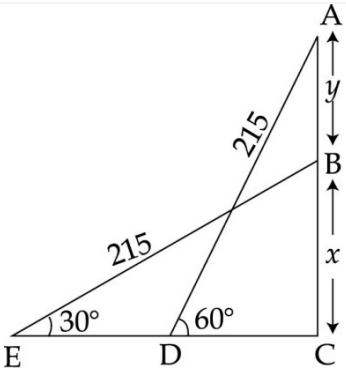


Let A and B be two positions of the balloon when the string makes an angle of 60° and 30° respectively.

In Δ BCE, $\frac{BC}{CE}=\sin 30°$

$$\frac{x}{215}=\frac{1}{2}$$

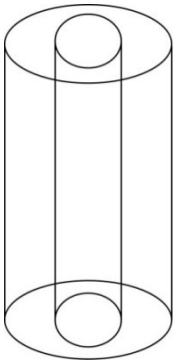
$$\Rightarrow x=\frac{215}{2}=107.5 \text{ m}$$

Also, in Δ ACD, $\frac{AC}{AD}=\sin 60°$

$$\frac{x+y}{215}=\frac{\sqrt{3}}{2}$$

4

		$2x+2y=215\sqrt{3}$ $215+2y=215\sqrt{3}$ $2y=215\ (\sqrt{3}\ -1)$ $y=\frac{215\left(\sqrt{3}-1\right)}{2}\text{m}$ Hence the height of the balloon when elevation was 30° is 107. 5m and when elevation was 60°, it is $(x+y)\ =\frac{215}{2}\sqrt{3}\text{ m}$	
	27	(i) P (at least 87) $=\frac{9}{52}$ (ii) P (less than 23) $=\frac{11}{52}$	4
	28	Using mid-section formula, the coordinates of mid-point of diagonal AC are $x_1=\frac{-2+4}{2}=1$ $y_1=\frac{-1+3}{2}=1$ i.e $(x_1, y_1)\ =\ (1, 1)$ Also, the coordinates of mid-point of diagonal BD are, $x_2=\frac{1+1}{2}=1$ $y_2=\frac{0+2}{2}=1$ i.e $(x_2, y_2)\ =\ (1, 1)$ Clearly, the points (x_1, y_1) and (x_2, y_2) are the same $\Rightarrow$ ABCD is a ||gm, as the diagonals bisect each other Now, area of ||gm ABCD $=$ ar ΔABC + ar ACD $=$ 2 arΔ ABC [diagonal divides a $\|\|$ gm into two $\cong\Delta$'s] now, ar ABC $=\frac{1}{2}\ [-2(0-3)\ +1(3+1)\ +4(-1-0)]$ $=\frac{1}{2}\ [6+\cancel{4}-\cancel{4}\ ]$ $=$ 3 sq. units $\therefore$ ar ||gm ABCD $=2\times3=6$ sq. units.	4
	29	Surface area $=\ 2\pi r_1 h+2\pi r_2 h+\ \pi\ r_2^2-\pi\ r_1^2=943\text{ cm}^2$ Total area to be painted $=50\times943=47150\text{ cm}^2$ Value: child labour. Exising law should be honestly implemented.	4

	30	Area of circular park $= \frac{27720}{5} = 5544$ $\pi R^2 = 5544$ $R = 42$ cm Area of raod $= \frac{10780}{3.5} = 3080$ Area of road $= \pi R^2 - \pi r^2 = 3080$ $1764 - r^2 = 980$ $r = 28$ cm Total length of fence $= 2 \times \frac{22}{7} \times (42 + 28) = 440$ m Cost of fencing $= 924$	4
	31	External diameter $= 16$ cm External radius 'R' $= 8$ cm Internal diameter $= 14$ cm Internal radius 'r' $= 7$ cm Height of vessel $= 42$ cm $= h$ Volume of cork dust = (Volume of External Cylinder) − (Volume of Internal Cylinder)  $= \pi R^2 h - \pi r^2 h$ $= \pi h(R^2 - r^2)$ $= \frac{22}{7} \times 42 \times (8^2 - 7^2)$ $= 1980$ cm^3	4
		-o0o0o0o-	